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Set membership estimation of day-ahead microgrids scheduling

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Abstract— **The net power output of a MicroGrid (MG) is often scheduled using optimization-based strategies. Recently, the new figure of the Aggregator (AG) has been introduced with the role of intermediate broker in the energy market, efficiently managing the interaction between a cluster of MGs and the system operators. To do that, the AG needs models of the related MGs to estimate both their power absorption/production, as a function of the energy prices, and the corresponding uncertainty ranges accounting for non-dispatchable generators and loads. To protect the MGs internal information and to reduce the complexity of the AG decision-making process, the problem of deriving these models from data is considered here. In order to cope with the problem nonlinearity and to quantify the uncertainty range, a nonlinear Set Membership approach is applied, and a new tuning method is described. The potentials of the proposed approach are tested with data obtained from a realistic MG model.**

I. Introduction

The power grid is shifting from the current centralized paradigm to a distributed framework where many autonomous elements, the *prosumers*, interact by producing and/or consuming power. Different kinds of prosumers exist (e.g. smart buildings, micro-grids, virtual power plants), all being clusters of different units that can generate, store, and consume electricity. Here, for the sake of generality, we consider the concept of micro-grids (MG), i.e. clusters equipped both with generators (e.g. micro-generators, renewable sources and storage units), storage (batteries) and loads, possibly controllable to some extend (e.g. curtailing and shifting loads). In principle, MGs can work either in isolated or in grid-connected modes, and these two conditions are characterized by different challenges [1], [2]. Regarding the grid-connected mode, each MG must interact with the system operator to trade its energy services on the market. This is clearly impractical as the number of connected MGs grows. In order to efficiently manage the interaction of a large number of MGs with the system operator, a new market figure has been recently defined, the Aggregator (AG) [3]. The AG serves as a broker in the energy market, operating as intermediary between its associated prosumers and the system operator. As reported in [4], the AG operations are characterized by two main tasks:

1) The AG participates in the day-ahead market for the definition of the energy prices, proposing supply/demand energy bids regarding its aggregated MGs.

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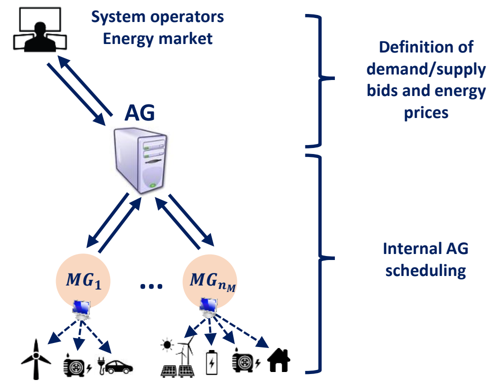

Fig. 1: Sketch of the considered energy market scenario.

2) After the day-ahead energy prices have been agreed upon, the AG interacts with its aggregation to optimally plan their internal operations.

In Figure 1, a sketch of the AG interactions is depicted. In principle, the AG operations involve large optimization problems that, if approached in a centralized way, would require the knowledge by the AG of all internal MGs' information, e.g. generators characteristics, internal management strategies and load consumption trends, as considered in [5]. To avoid privacy issues and to reduce the complexity of the AG decision making problem, a distributed optimization strategy is proposed in [6], where the AG and each MG iteratively solve simpler optimization problems. This approach effectively copes with the second task, when the energy prices have been already defined, however it is impractical for the first one, when the prices must be negotiated between the AG and the system operator. For this task, to optimize the trading outcome, the AG needs to estimate how much power its aggregation is going to produce/absorb based on the energy prices being traded, such that it can submit the supply/demand bids in the day-ahead market.

In this paper, we focus on this estimation problem, and pursue the idea of deriving a model of each MG from historical data of produced power vs. energy prices, which are available to the AG. We assume that the AG has no information on the layout of each MG and on the power trends of the non-dispatchable units (e.g. renewable sources and loads). Our first novel contribution is the use of a nonlinear Set Membership (SM) estimation approach [7] to overcome these problems. This identification technique provides both a nominal estimate and guaranteed uncertainty bounds. This is a significant advantage, since the AG can have not only a nominal estimate of the MG power trend as a function of the energy prices, but also certified power deviation bounds, accounting for the uncertainty. An important feature of the proposed method is that it requires rather mild assumptions on the uncertain components of produced/consumed power, which are simply required to be bounded. This suits very well the considered application, since the trends of loads and renewable sources, although unknown to the AG, are for sure bounded by their maximum power production/absorption values. The considered SM technique features tuning parameters that have to be chosen by the user: the assumed Lipschitz constant γ of the function to be learned, and the bound ϵ on the additive uncertainty. Without any a-priori knowledge on the system, the tuning of (γ, ϵ) might be time-consuming, see [7] for a discussion. As an additional contribution of this paper, we propose an optimization-based tuning procedure for (γ, ϵ) that aims to minimize the uncertainty interval associated with the nominal predicted power output, which is the most important feature in the considered application context. This paper represents a first study to evaluate the applicability of the proposed method, its accuracy and conservativeness.

II. Problem formulation

An AG, to interact with the system operator in the day-ahead market bidding process, must know the power that its aggregation of MGs is going to absorb/produce based on the daily trend of the energy prices. Indeed, each MG schedules the power set-points for its generating units and controllable loads, aiming to maximize its internal profit, considering also the effect of the non-dispatchable units, as in [2]. Therefore, considering the standard 15-minutes sampling interval, for each MG we can express the scheduled power output as:

$$y(t, \mathbf{p_e}) = f_t(\mathbf{p_e}) + d(t),\ t = 1, \dots, T, \tag{1}$$

where t is the discrete time variable, $T = 96$ is the number of considered time instants (i.e. one full day), $\mathbf{p_e} = \{p_e(1), \dots, p_e(T)\}^\top \in \mathbb{R}^T$ is the course of energy prices at 15-min intervals along the day ($^\top$ denotes the matrix transpose operator and bold-faced quantities denote vectors). Finally, $d(t)$ is an additive disturbance term, accounting for the effect of the non-dispatchable units and loads. For notational simplicity, in the remainder we consider a single MG; the whole approach presented in this paper can be then applied in parallel to all the MGs associated with the aggregator. The AG needs a model of the form (1) for each MG, i.e. an estimate $\tilde{y}(t, \mathbf{p_e}) \approx y(t, \mathbf{p_e})$. We propose to learn a separate model for each time instant. We assume that a finite data-set is available, given by the two sequences:

$$\begin{array}{rcl} Y_t & = & \{\tilde{y}^{(1)}(t, \tilde{\mathbf{p}}_\mathbf{e}^{(1)}), \dots, \tilde{y}^{(N)}(t, \tilde{\mathbf{p}}_\mathbf{e}^{(N)})\} \\ P_e & = & \{\tilde{\mathbf{p}}_\mathbf{e}^{(1)}, \dots, \tilde{\mathbf{p}}_\mathbf{e}^{(N)}\} \end{array} \tag{2}$$

where $\tilde{x}$ denotes a sampled data point of the generic variable x, $\tilde{y}^{(j)}(t)$ is the observed power that the MG provides at instant t in response to the energy price sequence $\tilde{\mathbf{p}}_\mathbf{e}^{(j)}$, and N is the total number of data points. Note that, due to the mentioned uncertainty in non-dispatchable generators and non-controllable loads, for any $\tilde{\mathbf{p}}_\mathbf{e}^{(j)} = \tilde{\mathbf{p}}_\mathbf{e}^{(k)}$, $j \neq k$, we can have $\tilde{y}^{(j)}(t) \neq \tilde{y}^{(k)}(t)$.In the described scenario, our goal is summarized as follows.

Problem 1. From the available data set Y_t, P_e, derive $T = 96$ approximating functions $f_t : \mathbb{R}^T \to \mathbb{R}$ such that:

$$\hat{y}(t, \mathbf{p_e}) = \hat{f}_t(\mathbf{p_e}),\ t = 1, \dots, T. \tag{3}$$

Moreover, for each considered t, derive two additional functions, $\bar{f}_t$ and $\underline{f}_t$, and an additive uncertainty bound, ϵ_t, providing guaranteed upper and lower bounds on $y(t)$:

$$\underline{f}_t(\mathbf{p_e}) - \epsilon_t \leq y(t, \mathbf{p_e}) \leq \bar{f}_t(\mathbf{p_e}) + \epsilon_t,\ t = 1, \dots, T. \tag{4}$$

The estimators $\hat{f}_t(\mathbf{p_e})$ and the guaranteed uncertainty intervals $\bar{f}_t(\mathbf{p_e}) - \underline{f}_t(\mathbf{p_e}) + 2\epsilon_t$, $t = 1, \dots, T$, could be then used by the AG in the day-ahead bidding process. This is part of ongoing research work not treated in this paper, where we focus instead on **Problem 1**. In Section IV we propose the use of SM identification to solve this problem, together with a novel tuning procedure for the identification routine. As anticipated in the Introduction, we consider here synthetic data, which we produced by simulating the optimization problem that a real MG would solve to schedule its elements. This allows us to evaluate two aspects: 1) whether at a theoretical level the working assumptions of the considered SM technique are satisfied, and 2) the estimation accuracy that can be achieved in an ideal scenario. We describe next the employed MG model.

III. Microgrid optimal scheduling model

We consider a generic MG equipped with n_g dispatchable generators (e.g. thermal engines), n_b batteries and n_{res} non-dispatchable generating units, typically based on renewable energy sources. Moreover, it is assumed that n_{ndl} non-dispatchable loads, n_{cl} curtailing loads and n_{sl} shifting loads are present. The active power absorption of the non-dispatchable loads can not be modified; on the contrary, the curtailing loads allow the MG to reduce its consumption in some predefined periods, while the shifting loads are characterized by a fixed energy demand which must be satisfied within some times constraints, giving some scheduling freedom. The variables and parameters of the MG elements are described in Table I.

The generation units are assumed to instantaneously follow the power set-points; this is reasonable with the assumed 15-minute sampling interval. As a convention, the powers are defined as positive if they are generated, while they are negative if absorbed. Considering the fuel-based generators and the batteries, their output power at time step t is constrained by the capability limits. Therefore, $\forall j \in \{1, \dots, n_g\}$ and $\forall i \in \{1, \dots, n_b\}$, it follows

$$\underline{u}_j^g \leq u_j^g(t) \leq \bar{u}_j^g, \qquad \underline{u}_i^b \leq u_i^b(t) \leq \bar{u}_i^b \tag{5}$$

The state of charge (SOC) of the batteries is modelled as a pure integrator and it is bounded by predefined limits; for the sake of simplicity, the charging and the discharging

TABLE I: MG optimization variables and parameters

Symbol	Description
u^g	Micro-generator active power set-point [kW]
u^b	Battery active power set-point [kW]
s^b	State of charge (SOC) [%]
u^{sl}	Shifting load active power output [kW]
u^{cl}	Curtailing load effective active power output [kW]
d^{ndl}	Non-dispatchable load active power output [kW]
d^{res}	Renewable sources active power output [kW]
y^{mg}	MG active power output [kW]
$\bar{u}^g$, $\underline{u}^g$	Micro-generator power limits [kW]
$\bar{u}^b, \underline{u}^b$	Battery power limits of battery [kW]
$\bar{s}^b, \underline{s}^b$	SOC limits [%]
C^b	Battery capacity [kWh]
$\bar{u}^{sl}$, $\underline{u}^{sl}$	Shifting loads power limits [kW]
$\bar{e}^{sl}$	Shifting loads energy demand [kWh]
$\bar{\tau}^{sl}$, $\underline{\tau}^{sl}$	Shifting loads time limits [h]
d^{cl}	Curtailing load predefined active power output [kW]
$\Delta\bar{u}^{cl}$	Max curtailing loads power reduction [kW]
$\bar{\tau}^{cl}$, $\underline{\tau}^{cl}$	Curtailing loads time limits [h]
c^{cl}	Curtailing load cost [€/kWh^2]
c^b	Battery usage cost [€/kWh^2]
a^g, b^g, c^g	Micro-generator cost coefficients [€/kWh^2, €/kWh, €]
p_e	Energy price [€/kWh]

efficiencies are here neglected. Moreover, it is supposed that the SOC at the end of the day must be equal to the one at the beginning, in order to start the next day with the same initial storage conditions, see e.g. [8]. Therefore, $\forall j \in \{1, \dots, n_b\}$,

$$s_j^b(t+1) \;=\; s_j^b(t) \;-\; 100\,\frac{\tau}{C_j^b}\,u_j^b(t) \tag{6a}$$

$$\underline{s}_j^b \;\leq\; s_j^b(t) \;\leq\; \bar{s}_j^b, \quad s_j^b(T) = s_j^b(0) \tag{6b}$$

where $\tau = 0.25$ hours is the employed sampling interval. The shiftable, or deferrable, loads are usually characterized by active power limits and, moreover, they can be activated just in a predefined time range where a specific amount of energy demand must be satisfied. Therefore, the following constraints are imposed, $\forall j \in \{1, \dots, n_{sl}\}$,

$$\underline{u}_j^{sl} \;\leq\; |u_j^{sl}(t)| \;\leq\; \bar{u}_j^{sl} \quad \forall\, t \in \{\underline{\tau}_j^{sl}, \bar{\tau}_j^{sl}\} \tag{7a}$$

$$u_j^{sl}(t) \;=\; 0 \qquad \forall\, t \notin \{\underline{\tau}_j^{sl}, \bar{\tau}_j^{sl}\} \tag{7b}$$

$$\sum_{\forall\, t \in \{\underline{\tau}_j^{sl}, \bar{\tau}_j^{sl}\}} |u_j^{sl}(t)|\,\tau = \bar{e}_j^{sl} \tag{7c}$$

The curtailing loads allows the MG operator to reduce their consumption in certain time slots, with respect to the predefined fixed demand d^{cl}, within a maximum reduction limit,$\forall j \in \{1, \dots, n_{cl}\}$,

$$|d_j^{cl}(t) \;-\; u_j^{cl}(t)| \;\leq\; \Delta\bar{u}^{cl} \qquad \forall\, t \in \{\underline{\tau}_j^{cl}, \bar{\tau}_j^{cl}\} \tag{8a}$$

$$|d_j^{cl}(t) \;-\; u_j^{cl}(t)| \;=\; 0 \qquad \forall\, t \notin \{\underline{\tau}_j^{cl}, \bar{\tau}_j^{cl}\} \tag{8b}$$

Let us now collect for simplicity the sum of all active power setpoints of the dispatchable units (i.e. the MG decision variables) and the sum of active power values of non-dispatchable units (i.e. the disturbance variables) in two vectors, $\mathbf{u}$ and $\mathbf{d}$, respectively:

$$\begin{aligned}
\mathbf{u} &= [u(1), \dots, u(T)]^\top \in \mathbb{R}^T \\
u(t) &= \sum_{j=1}^{n_g} u_j^g(t) + \sum_{j=1}^{n_b} u_j^b(t) + \sum_{j=1}^{n_{sl}} u_j^{sl}(t) + \sum_{j=1}^{n_{cl}} u_j^{cl}(t) \\
\mathbf{d} &= [d(1), \dots, d(T)]^\top \in \mathbb{R}^T \\
d(t) &= \sum_{j=1}^{n_{ndl}} d_j^{ndl}(t) + \sum_{j=1}^{n_r} d_j^{res}(t)
\end{aligned}$$

We can now introduce the cost function to be minimized in the optimal scheduling computation (see Table I for the involved parameters):

$$\begin{aligned}
J = &\sum_{t=1}^{T} \underbrace{\sum_{j=1}^{n^g} (a_j^g \tau^2 (u_j^g(t))^2 + b_j^g\, \tau\, u_j^g(t) + c_j^g)}_{\alpha} + \\
&+ \sum_{t=2}^{T} \underbrace{\sum_{j=1}^{n^b} c_j^b \tau^2 \left(u_j^b(t) - u_j^b(t-1)\right)^2}_{\beta} + \\
&+ \sum_{t=1}^{T} \underbrace{\sum_{j=1}^{n^{cl}} c_j^{cl} \tau^2 (d_j^{cl}(t) \;-\; u_j^{cl}(t))^2}_{\eta} \\
&- \sum_{t=1}^{T} \tau \underbrace{(p_e(t)\; (u(t) + d(t)))}_{\gamma}
\end{aligned} \tag{9}$$

where all the costs/prices are multiplied by the sampling time τ, since they are commonly referred to the amount of energy generated/consumed. The fuel-generator cost α is modelled with a quadratic polynomial function, as common in the literature.Although batteries do not have an effective cost, the term β is expressed to penalize the square of the power variation to avoid excessive charges and discharges which may reduce the battery life. Since the load curtailing usually leads to some discomfort issues, their power reduction will be characterized by a cost expressed by the term η, as in [2]. Finally, the term γ indicates the net output power of the MG. In this study, the MG management system is supposed to solve the following optimization problem:

$$\begin{aligned}
&\min_{\mathbf{u}} \quad J \\
&\text{subject to} \quad (5)\text{ - }(8)
\end{aligned} \tag{10}$$

Note that (10) is a convex quadratic program (QP), for which a global minimizer can be efficiently computed. Moreover, under reasonable assumptions on the MG constraints and weights in the cost function, the problem is strictly convex and such a minimizer is unique. Let us denote with $\mathbf{u}^*(\mathbf{p_e}) = [u^*(1, \mathbf{p_e}), \dots, u^*(T, \mathbf{p_e})]^\top$ a global minimizer of (10). Then, by the internal power balance, the MG power output at each time t is given by:

$$y(t, \mathbf{p_e}) = u^*(t, \mathbf{p_e}) + d(t). \tag{11}$$

Remark 1: Note that the terms $d(t), t = 1, \dots, T$ do not affect the value $\mathbf{u}^*(\mathbf{p_e})$, since in (9) they contribute to an additive offset that doesn't depend on the optimization variables. Thus, the solution of (10) depends only on the predicted prices $\mathbf{p_e}$, and we can express (11) equivalently as:

$$y(t, \mathbf{p_e}) = \underbrace{f_t(\mathbf{p_e})}_{u^*(t,\mathbf{p_e})} + d(t), \qquad (12)$$

which is consistent with our problem setup, see (1). Moreover, by the implicit function theorem, strict convexity of (10) results in Lipschitz continuity of functions $f_t(\mathbf{p_e})$, $t = 1, \dots, T$. Finally, the additive term $d(t)$ accounts for all uncertain, non-controllable sources and loads, and it is a bounded quantity with generally unknown bounds, that can however be estimated from data.

Overall, the features highlighted in Remark 1 fit perfectly with the prior assumptions of the SM identification approach that we adopt, recalled in the next section.
Finally, to generate the data that we will use for the estimation procedures, we solve problem (10) with a finite number N of energy price courses $\tilde{\mathbf{p}}_{\mathbf{e}}^{(j)}$ and uncertain (e.g. random) values of $\tilde{d}(t)^{(j)}$, and store the resulting values of $\tilde{y}(t, \tilde{\mathbf{p}}_{\mathbf{e}}^{(j)})$ computed as in (12), for all $t = 1, \dots, T$ and $j = 1, \dots, N$. Then, we collect such data in the sequences Y_t, P_e, see (2).

IV. Identification procedure

We summarize the SM identification approach of [7], which we employ here to solve **Problem 1**, and introduce the new optimization-based tuning procedure we propose. For notational simplicity, we consider a fixed value of t and drop the notation $t = 1, \dots, T$, since it is implicit that the whole identification process has to be carried out for each value of t. Thus, we want to derive from the available data Y_t, P_e an approximation of function $f_t(\mathbf{p_e})$ and guaranteed upper and lower bounds. This is achieved in three steps, described in the following sub-sections.

Before proceeding further, we subdivide the available data as $Y_t = \{Y_{t,A}\,, Y_{t,C}\}$ and $P_e = \{P_{e,A}\,, P_{e,C}\}$, where the subscripts $_A$, $_C$ stand for approximation and calibration, respectively. Moreover, we also define the sets of integers $\mathcal{N}_A = \{1, \dots, N_A\}$ and $\mathcal{N}_C = \{N_A + 1, \dots, N_A + N_C\}$, where N_A and N_C are the cardinalities of $Y_{t,A}$ and $Y_{t,C}$.

A. First step: compute a preliminary approximating function

We first derive an estimate $f'_t \approx f_t$ using the approximation data set $Y_{t,A}$, $P_{e,A}$. In this step, one can choose the functional forms of f' among many existing possibilities (linear regression, polynomials, neural nets, etc.), as long as it enjoys Lipschitz continuity. Here, we resort to a linear regression:

$$f'_t(\mathbf{p_e}) = \hat{\theta}^\top \mathbf{p_e}, \qquad (13)$$

whose parameters $\hat{\theta}$ are computed with a ℓ_1-norm regularization approach:

$$\hat{\theta} = \underset{\theta}{\operatorname{argmin}}\{ \sum_{\forall i \in \mathcal{N}_A} ||\tilde{y}_t^{(i)} - \theta^\top \tilde{\mathbf{p}}_{\mathbf{e}}^{(i)}||_2^2 + \lambda \|\theta\|_1 \}, \qquad (14)$$

where the scalar $\lambda > 0$ is a tuning parameter.

B. Second step: define the residual function, collect the related data points, and compute the feasible parameter region

Using the obtained preliminary approximation (13), we define the residual function Δ_t as:

$$\Delta_t(\mathbf{p_e}) = f_t(\mathbf{p_e}) - \hat{\theta}^\top \mathbf{p_e}. \qquad (15)$$

Then, we can alternatively write (1) as:

$$y(t, \mathbf{p_e}) = \hat{\theta}^\top \mathbf{p_e} + \Delta_t(\mathbf{p_e}) + d(t). \qquad (16)$$

We will now focus on the derivation of an approximating function $\hat{\Delta}_t \approx \Delta_t$ using the nonlinear SM approach of [7]. The following prior knowledge is available:

1) Lipschitz continuity of Δ_t (thanks to Lipschitz continuity of both f_t and f'_t, see Remark 1 and (13)):

$$|\Delta_t(\mathbf{p_e}^{(j)}) - \Delta_t(\mathbf{p_e}^{(k)})| \leq \gamma_t \|\mathbf{p_e}^{(j)} - \mathbf{p_e}^{(k)}\|_2, \quad \forall \mathbf{p_e}^{(j)},\, \mathbf{p_e}^{(k)} \in P \qquad (17)$$

2) Boundedness of $d(t)$ (see Remark 1):

$$|d(t)| \leq \epsilon_t \qquad (18)$$

where $P \subset \mathbb{R}^T$ is a compact set containing all the possible energy price courses over the day. The positive scalars γ_t, ϵ_t in (17)-(18) are not known, and shall be estimated from data as well. To do so, we start by computing the *feasible parameter region* $\mathcal{B} \subset \mathbb{R}^+ \times \mathbb{R}^+$. Namely, this is the set of (γ_t, ϵ_t) pairs such that the prior knowledge is consistent with the available data. Using our data-set, we first obtain samples of the residuals $\tilde{\Delta}_t^{(i)}$ as:

$$\tilde{\Delta}_t^{(i)} = \tilde{y}_t^{(i)} - \hat{\theta}^\top \tilde{\mathbf{p}}_{\mathbf{e}}^{(i)} \qquad i = 1, \dots, N \qquad (19)$$

Then, for a given pair (γ_t, ϵ_t), let us define the following functions:

$$\bar{\Delta}_t(\mathbf{p_e}) \doteq \min_{\forall j=1,\dots,N} (\tilde{\Delta}_t^{(j)} + \epsilon_t + \gamma_t \|\mathbf{p_e} - \tilde{\mathbf{p}}_{\mathbf{e}}^{(j)}\|_2) \qquad (20)$$

$$\underline{\Delta}_t(\mathbf{p_e}) \doteq \min_{\forall j=1,\dots,N} (\tilde{\Delta}_t^{(j)} - \epsilon_t - \gamma_t \|\mathbf{p_e} - \tilde{\mathbf{p}}_{\mathbf{e}}^{(j)}\|_2) \qquad (21)$$

Now, we can compute $\mathcal{B}$ by exploiting the following result (*Theorem 1* in [7]):
A necessary and sufficient condition for the prior knowledge to be validated is:

$$\bar{\Delta}_t(\tilde{\mathbf{p}}_{\mathbf{e}}^{(i)}) > \tilde{\Delta}_t^{(i)} - \epsilon_t \quad \forall i \in \mathcal{N}_A \cup \mathcal{N}_C \qquad (22)$$

The feasible parameter set is defined as:

$$\mathcal{B} = \{(\gamma_t, \epsilon_t) : \text{condition (22) holds}\} \qquad (23)$$

Note that the set $\mathcal{B}$ is unbounded, since it is always possible to satisfy condition (22) with large enough values of γ_t and/or ϵ_t. On the other hand, it is of interest to compute the "lower boundary" $\underline{\gamma}_t(\epsilon_t)$:

$$\underline{\gamma}_t(\epsilon_t) \;=\; \begin{array}{c} \arg\min_{\gamma} \gamma \\ \text{s.t.} (\gamma, \epsilon_t) \in \mathcal{B} \end{array} \qquad (24)$$

Fig. 2 presents a qualitative example of set $\mathcal{B}$. We employ such a curve to select suitable estimates $\hat{\gamma}_t, \hat{\epsilon}_t$ and eventually obtain our approximating function and error bounds, as described next.

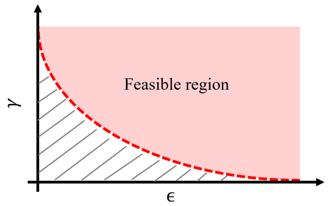

Fig. 2: Example of γ_t-ϵ_t feasible parameter set

C. Third step: select an estimate of the Lipschitz constant and error bound, and derive the approximating function $\hat{f}_t$

To derive the wanted approximating functions, one has to select a pair $(\hat{\gamma}_t, \hat{\epsilon}_t)$ inside the set $\mathcal{B}$. Assuming that such estimates have been chosen, then the corresponding functions $\bar{\Delta}_t(\mathbf{p_e})$ and $\underline{\Delta}_t(\mathbf{p_e})$ in (20)-(21) represent the upper and lower bounds of $\Delta_t(\mathbf{p_e})$. Moreover, if the prior knowledge is valid, the interval $\left[\underline{\Delta}(\mathbf{p_e}),\, \bar{\Delta}_t(\mathbf{p_e})\right]$ is guaranteed to include the true function $\Delta(\mathbf{p_e})$ [7]. As a result, for a given price prediction $\mathbf{p_e} \in P$, the output $y(t, \mathbf{p_e})$ (16) is guaranteed to be bounded by the following functions:

$$\underline{y}(t, \mathbf{p_e}) \leq y(t, \mathbf{p_e}) \leq \bar{y}(t, \mathbf{p_e}) \tag{25}$$

$$\begin{aligned} \bar{y}(t, \mathbf{p_e}) &= \theta^\top \mathbf{p_e} + \bar{\Delta}_t(\mathbf{p_e}) + \epsilon_t \\ \underline{y}(t, \mathbf{p_e}) &= \theta^\top \mathbf{p_e} + \underline{\Delta}_t(\mathbf{p_e}) - \epsilon_t \end{aligned} \tag{26}$$

and the estimated uncertainty interval is given by

$$\bar{y}(t, \mathbf{p_e}) - \underline{y}(t, \mathbf{p_e}) = \bar{\Delta}_t(\mathbf{p_e}) - \underline{\Delta}_t(\mathbf{p_e}) + 2\epsilon_t > 0 \tag{27}$$

In the literature, it is often the case that the pair (γ, ϵ) is selected in the feasible region using some prior knowledge on the system to be identified. When such a knowledge is not available, one has to choose these parameters using the available input-output data, which can be a time-consuming task. To this end, a possible approach is to choose the optimal parameters in the feasible region by minimizing the identification error with respect to the calibration data. This method can achieve a small approximation error on average, however the resulting uncertainty interval can be rather large. On the other hand, in the application considered here it is more reasonable to select estimates $\hat{\gamma}_t, \hat{\epsilon}_t$ such that the uncertainty interval (27) is as small as possible. Therefore, we propose to choose $(\hat{\gamma}_t, \hat{\epsilon}_t)$ by solving the following optimization problem, which involves the calibration dataset:

$$(\hat{\gamma}_t, \hat{\epsilon}_t) = \underset{(\gamma,\epsilon)\in\mathcal{B}}{\operatorname{argmin}} \{ \sum_{\forall j \in \mathcal{N}_C} (\, \bar{y}(t, \tilde{\mathbf{p}}_\mathbf{e}^{(j)}) - \underline{y}(t, \tilde{\mathbf{p}}_\mathbf{e}^{(j)})\,)^2 \} \tag{28}$$

Having computed the parameters $(\hat{\gamma}_t, \hat{\epsilon}_t)$, the wanted approximating function (see (3)) is given by:

$$\hat{y}(t, \mathbf{p_e}) = \hat{f}_t(\mathbf{p_e}) \quad = \quad \theta^\top \tilde{\mathbf{p}}_\mathbf{e} + \frac{\bar{\Delta}_t(\tilde{\mathbf{p}}_\mathbf{e}) + \underline{\Delta}_t(\tilde{\mathbf{p}}_\mathbf{e})}{2}$$

while the guaranteed power bounds can be computed as in (26). Function $\hat{f}_t(\mathbf{p_e})$ is the central (or optimal) approximation, i.e. it provides the smallest worst-case approximation error for the given data-set and chosen $(\hat{\gamma}_t, \hat{\epsilon}_t)$, equal to half the uncertainty bound (radius of information, see e.g. [7]).

V. Numerical results

The numerical results have been carried out considering a MG composed of different generation units and loads, whose parameters are reported in Table II. Considering the identification data set, the non-dispatchable power trends of load absorption and of renewable sources production are reported in Figure 3(a) and 3(b), respectively. The daily prices profiles are reported in Figure 3(c): their trends are defined considering the real daily prices of the Italian Day-Ahead Market (MGP), which can be found in [9]. In Figure 3(d) the resulting sampled data of MG output power are depicted, which have been computed according to the optimization framework described in Section III. As described in Section IV, the identification data set is divided in two different sets as follows: $N_A = 30$ approximation data, and $N_C = 120$ calibration data. The identification process is performed as described in Section IV and the optimal tuning parameters $(\hat{\gamma}_t, \hat{\epsilon}_t)$ are computed. Then, the performance of the identification process are evaluated using an additional *validation data* set, denoted as $\mathcal{N}_\mathcal{V}$, with $N_V = 60$ data points. Before proceeding to the description of the numerical results, the following variables are defined $\mathbf{y}(\mathbf{p_e})$, $\hat{\mathbf{y}}(\mathbf{p_e})$, $\bar{\mathbf{y}}(\mathbf{p_e})$, $\underline{\mathbf{y}}(\mathbf{p_e})$, which express the daily MG output power, the corresponding estimate and the upper and lower bounds, respectively. The following variable is also introduced, expressing the average output power of the MG

$$y_m(\mathbf{p_e}) = \sum_{t=1}^{T} y(t, \mathbf{p_e})$$

To properly evaluate the achieved performance, the following indexes are computed $\forall i \in \mathcal{N}_\mathcal{V}$

$$IE^{(i)} = \frac{||\mathbf{y}(\mathbf{p}_\mathbf{e}^{(i)}) - \hat{\mathbf{y}}(\mathbf{p}_\mathbf{e}^{(i)})||_2}{\sqrt{T}}, \qquad IE_{\%}^{(i)} = \frac{100\, IE^{(i)}}{y_m(\mathbf{p}_\mathbf{e}^{(i)})}$$

$$MD^{(i)} = \frac{||\bar{\mathbf{y}}(\mathbf{p}_\mathbf{e}^{(i)}) - \underline{\mathbf{y}}(\mathbf{p}_\mathbf{e}^{(i)})||_\infty}{2}, \quad MD_{\%}^{(i)} = \frac{100\, MD^{(i)}}{y_m(\mathbf{p}_\mathbf{e}^{(i)})}$$

where IE corresponds to the identification error computed for the whole daily period, while MD represents the maximum possible deviation of the output with respect to the estimate, considering the computed power bounds. For the sake of completeness, also the values normalized with respect to the average output power y_m are computed, i.e. $IE_\%$ and $MD_\%$. The achieved performance indicators have been computed considering the whole validation data set $\mathcal{N}_\mathcal{V}$ and the results are reported in Table III. The reported results witness the effectiveness of the proposed approach both in terms of reduced identification error and of capability to provide guaranteed bounds for the MG output power. To better highlight the advantage of the proposed SM approach, the results obtained with two exemplary validation data trends are presented in Figure 4. Precisely, Figure 4(a)(b) shows a case when a small identification error is achieved, $IE = 2.16\, kW$; it is worth noticing that the real MG output power is fully contained in the computed bounds, see Figure 4(b). On the other hand, Figure 4(c)-(d) show a case where

TABLE II: MG parameters and constraints

Generators	$(\underline{u}^g, \bar{u}^g)$	a^g	b^g	c^g
u_1^g	$(20, 100)$	$1.25e-5$	$1.25e-3$	$3e-2$
u_2^g	$(20, 50)$	$2.25e-5$	$7.5e-4$	$3.5e^{-3}$
Batteries	$(\underline{u}^b, \bar{u}^b)$	$(\underline{s}^b, \bar{s}^b)$	C^b	c^b
u_1^b	$(-50, 50)$	$(0.2, 0.8)$	40	$2.5e^{-2}$
u_2^b	$(-40, 40)$	$(0.2, 0.8)$	50	$2.5e^{-2}$
Shifting loads	$(\underline{u}^{sl}, \bar{u}^{sl})$	$(\underline{\tau}^{sl}, \bar{\tau}^{sl})$	$\bar{e}^{sl}$	$-$
u_1^{sl}	$(0, 20)$	$(9, 12)$	20	$-$
u_2^g	$(0, 20)$	$(16, 19)$	20	$-$
Curtailing loads	$\lvert d^{cl}(\forall t)\rvert$	$(\underline{\tau}^{cl}, \bar{\tau}^{cl})$	Δu^{cl}	c^{cl}
u_1^{cl}	20	$(12, 16)$	10	$3.13e-2$

TABLE III: Identification performances

	mean	min	max
IE	4.79 kW	2.16 kW	8.20 kW
$IE_{\%}$	7.1%	3%	14.7%
MD	14.62 kW	11.62 kW	20.17 kW
$MD_{\%}$	20%	17.3%	30%

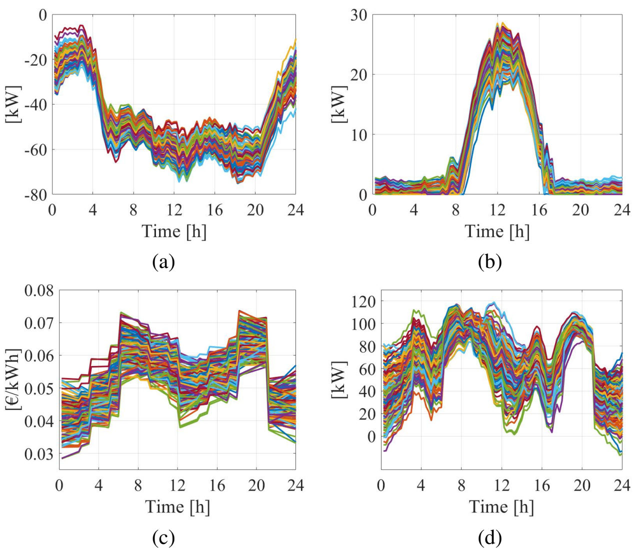

Fig. 3: Identification data: (a) Non-dispachable load power trends, (b) Renewable energy sources power trends, (c) Energy prices trends, (d) MG output power trends

a larger estimation error is obtained, with $IE = 8.2\,kW$. This is because the price trend used for this second case was quite different with respect to the identification data shown in Figure 3(c). Notwithstanding the estimation error, also in this second case the true power output is still contained in the computed bounds, see Figure 4(d). The knowledge of such bounds can be thus effectively used to account for the uncertainty in the day-ahead energy price definition process.

VI. Conclusions

The problem of deriving from data an estimate of the price-based power scheduling of microgrids has been considered. Aggregators can use the obtained models to optimize their trading process with the system operator, by better predicting the power output of each one of the associated microgrids as a function of the predicted energy prices. The proposed method is based on Set Membership theory and provides also guaranteed error bounds, which can be exploited by the aggregator to improve robustness of its decisions. The next steps in this research will be the use of real-world data to learn and validate the wanted microgrid models, and the study of how such models can be best exploited in the energy trading process.

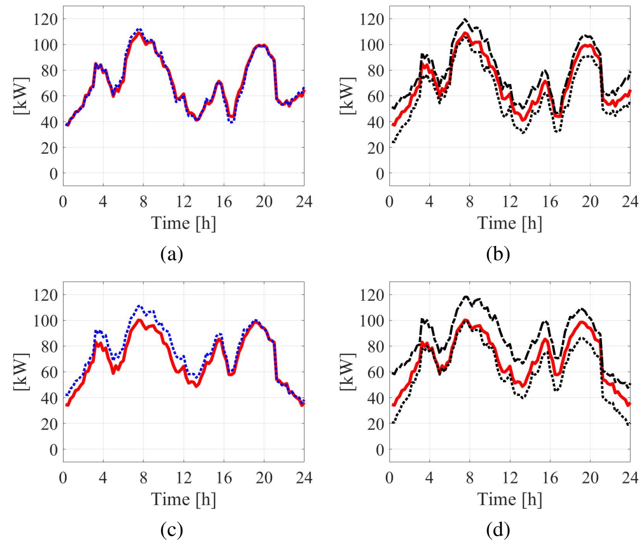

Fig. 4: Example of validation outcome with small estimation error and large estimation error: (a)&(c) real MG output power trend (solid line), estimated MG output power (dotted line), (b)&(d) higher power bound (dashed line), lower power bound (dotted line), real MG output power (solid line)

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